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# Teleportation of GHZ-States in QED-Cavities without the Explicit Bell-State Measurement

**W.B. Cardoso**

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**Abstract** In this paper we show how to teleport $N$-entangled states of $N$-QED-cavities without Bell-state measurements. The method has potential application in teleportation schemes requiring multipartite entanglements. The success probability and fidelity of the teleportation are also considered.

**Keywords** Quantum Teleportation · GHZ-states · Bell-state measurement

## 1 Introduction

Teleportation of quantum states, introduced in the pioneer work by Bennett et al. [4], is a tool having potential applications in quantum computation [7, 13, 17, 19] and in quantum cryptography [3, 12, 20]. The interest comes from the fact that the speed of information process is much greater than that in the conventional case. Since a quantum computer can factorize numbers with very high efficiency its construction naturally needs the development of quantum cryptography. Various experimental results on teleportation [5, 6, 11, 16] have inspired the studies in quantum computation, one of its main purposes being the acceleration of computation in small areas of space [19]. It is argued in the literature that teleportation of entangled states is the route to increasing quantum computation [1].

Entangled state, in special entanglement of multi-parties, is of fundamental interest in quantum many-body theory [21] and makes quantum information processing and distillation protocol more efficient than that relying on entanglement using only two-parties [2, 18]. The most popular entanglement of tripartite systems is the GHZ state [14]

$$|\Psi_{\text{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle). \tag{1}$$

When concerning with multipartite entanglements, the role analogous to the EPR and GHZ states is played by the so-called Schrödinger-cat states [24]; such an example is given

W.B. Cardoso (✉)
Instituto de Física, Universidade Federal de Goiás, 74.001-970 Goiânia, GO, Brazil
e-mail: wesleybcardoso@gmail.com

by

$$|\Psi_{\text{cat}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1|0\rangle_2|0\rangle_3\ldots|0\rangle_N + |1\rangle_1|1\rangle_2|1\rangle_3\ldots|1\rangle_N). \tag{2}$$

It is implied that, due to the large number $N$ of the particles involved, states $(|0\rangle_1|0\rangle_2 |0\rangle_3\ldots|0\rangle_N)$ and $(|1\rangle_1|1\rangle_2|1\rangle_3\ldots|1\rangle_N)$ describe different states of a macroscopic object. Consequently, the state (2) applies to a coherent superposition of macroscopically distinct states of a large object, similarly to the superposition of "dead" and "alive" states of a cat, introduced by Schrödinger [24]. Nowadays, these states consist of relatively few particles and become the object of many studies owing to the appearance of experimental possibilities for their creation. It has been argued, after some successful experimental results, that it should be relatively simple to entangle more than two atomic or field-mode samples [16] accordingly, it may be more practical and robust to weakly entangle many systems rather than strongly entangle a few of them.

Some of the potentials applications of entangled states are: (i) in fundamentals of quantum mechanics; (ii) metrology; (iii) quantum information; (iv) quantum teleportation, etc.

To accomplish quantum teleportation one is initially concerned with two separated locals, named Alice and Bob, described by an entangled quantum state, usually explained in terms of a nonlocal quantum channel of EPR [10] and measured in the Bell basis [4]. In the present case, we will no longer make an explicit Bell state measurement, since our scheme is so simple—using QED-cavities and single ionization detectors to detect $N$ atoms in the excited states. $N$ also stands for the number of the entangled cavity-states to be teleported. In a previous scheme [15] the authors were concerned with the teleportation of two-particle and multipartite entangled states; the scheme manipulates the interaction of atoms with the modes of high-Q cavities. Pires et al. have also presented a proposal to teleport entanglements of zero- and one-photon states in a cavity-QED [22], using Bell-state measurement. In recent works, Zheng [25] and Cardoso et al. [8], have shown a teleportation of an unknown quantum state without using the Bell-state measurement. Recently, some schemes have been proposed for information concentration without Bell-state measurement [26, 27], useful for quantum communication. In this paper we present a generalized scheme to teleport $N$ entangled zero- and one-photon states in cavities-QED without the explicit use of the Bell-state measurement. The present work is an extension of the paper of Zheng [25] and of Cardoso et al. [8], now for a state-GHZ of $N$ $(N > 3)$ qubits.

## 2 Teleportation of GHZ States

Consider initially a GHZ-entangled state

$$|\psi\rangle_{\text{in}} = \alpha|1\rangle_D|1\rangle_E|1\rangle_F - \beta|0\rangle_D|0\rangle_E|0\rangle_F, \tag{3}$$

where $\alpha$ and $\beta$ are unknown coefficients. The subscripts $D$, $E$ and $F$ refer to the cavities $D$, $E$ and $F$, respectively. To get the requested teleportation the atoms are sent in the excited state $|e\rangle$ to interact with a pair of cavities and they should also be detected in excited state. We use the Jaynes-Cummings Hamiltonian; in resonance, the atom-field interaction is given by (interaction picture),

$$H_{\text{in}} = \hbar\lambda(a^{\dagger}\sigma_- + a\sigma_+), \tag{4}$$

where $\lambda$ is the atom-field coupling constant, $a^{\dagger}$ and $a$ are the creation and annihilation operators for the cavity modes, the $\sigma_+$ and $\sigma_-$ represent the Pauli operators.

Using the same sequence above, we will send the atom 1 through the cavity $A$ (initially in the vacuum state), leaving the state of the entire system (cavities $A$, $B$, $C$, and atom 1) to

$$|\varphi\rangle_1 = \frac{1}{\sqrt{2}}[|0\rangle_A|0\rangle_B|0\rangle_C|e\rangle_1 - i|1\rangle_A|0\rangle_B|0\rangle_C|g\rangle_1],$$

with the adjustment $gt = \frac{\pi}{4}$. Now when the atom 1 crosses the cavity $D$ and the state of the whole system can be represented by

$$\begin{aligned}|\varphi\rangle_2 = \frac{1}{\sqrt{2}}[&\alpha\cos(\sqrt{2}gt')|0\rangle_A|0\rangle_B|0\rangle_C|1\rangle_D|1\rangle_E|1\rangle_F|e\rangle_1\\ &- i\alpha\sin(\sqrt{2}gt')|0\rangle_A|0\rangle_B|0\rangle_C|2\rangle_D|1\rangle_E|1\rangle_F|g\rangle_1\\ &- \beta\cos(gt')|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F|e\rangle_1\\ &+ i\beta\sin(gt')|0\rangle_A|0\rangle_B|0\rangle_C|1\rangle_D|0\rangle_E|0\rangle_F|g\rangle_1\\ &- i\alpha\cos(gt')|1\rangle_A|0\rangle_B|0\rangle_C|1\rangle_D|1\rangle_E|1\rangle_F|g\rangle_1\\ &- \alpha\sin(gt')|1\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|1\rangle_E|1\rangle_F|e\rangle_1\\ &+ i\beta|1\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F|g\rangle_1].\end{aligned}$$

After the detection of the atom 1 in the excited state $|e\rangle_1$, the state $|\varphi\rangle_2$ collapses onto

$$\begin{aligned}|\varphi'\rangle_2 = \frac{1}{\sqrt{2}}[&\alpha\cos(\sqrt{2}gt')|0\rangle_A|0\rangle_B|0\rangle_C|1\rangle_D|1\rangle_E|1\rangle_F\\ &- \beta\cos(gt')|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F\\ &- \alpha\sin(gt')|1\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|1\rangle_E|1\rangle_F].\end{aligned}$$

For the adjustment $gt' = \frac{7\pi}{4}$ the system evolves to the approximated state

$$|\varphi\rangle_3 \cong [\alpha|1\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|1\rangle_E|1\rangle_F - \beta|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F].$$

Following the previous sequence, we let the atom-2 to cross the cavity $B$. After the atom-field interaction we find (with the adjustment $gt = \frac{\pi}{4}$)

$$\begin{aligned}|\varphi\rangle_4 = \frac{1}{\sqrt{2}}[&\alpha|1\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|1\rangle_E|1\rangle_F|e\rangle_2 - i\alpha|1\rangle_A|1\rangle_B|0\rangle_C|0\rangle_D|1\rangle_E|1\rangle_F|g\rangle_2\\ &- \beta|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F|e\rangle_2 + i\beta|0\rangle_A|1\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F|g\rangle_2].\end{aligned}$$

Next, the atom 2 crosses the cavity $E$ and, right away, this atom is detected. If we detect the atom 2 in the excited state $|e\rangle_2$, for an adjustment $gt' = \frac{7\pi}{4}$, the atom-field system is projected onto the state

$$|\varphi\rangle_5 \cong [\alpha|1\rangle_A|1\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|1\rangle_F - \beta|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F].$$

Finally, setting the atom 3 to cross the cavity $C$ we obtain the system projected onto the state

$$\begin{aligned}|\varphi\rangle_6 = \frac{1}{\sqrt{2}}[&\alpha|1\rangle_A|1\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|1\rangle_F|e\rangle_3 - i\alpha|1\rangle_A|1\rangle_B|1\rangle_C|0\rangle_D|0\rangle_E|1\rangle_F|g\rangle_3\\ &- \beta|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F|e\rangle_3 + i\beta|0\rangle_A|0\rangle_B|1\rangle_C|0\rangle_D|0\rangle_E|0\rangle_F|g\rangle_3],\end{aligned}$$

where the adjustment $gt = \frac{\pi}{4}$ was assumed. After the interaction with the cavity $F$ (with $gt' = \frac{7\pi}{4}$), the atom 3 should be detected in the excited state $|e\rangle_3$. In this way the state of the cavities can be represented by the approximated state

$$|\psi\rangle_T \cong [\alpha|1\rangle_A|1\rangle_B|1\rangle_C - \beta|0\rangle_A|0\rangle_B|0\rangle_C] \otimes |0\rangle_D|0\rangle_E|0\rangle_F. \quad (5)$$

Note that the cavities $D$, $E$ and $F$ are in the vacuum state, as they were initially for the cavities $A$, $B$ and $C$. Then the system composed by the cavities $A$, $B$ and $C$ is projected in the teleported state

$$|\psi\rangle_{\text{out}} \cong \alpha|1\rangle_A|1\rangle_B|1\rangle_C - \beta|0\rangle_A|0\rangle_B|0\rangle_C. \quad (6)$$

To conclude the teleportation we need of a classical channel [4] to inform the receiver whether he (Bob) has obtained the wanted teleported state after the detections of the three atoms (by Alice) in the excited state ($|e\rangle$). If that is not the case, one might start the teleportation procedure again.

## 3 Generalized Scheme

Next, to generalize the previous scheme teleporting the GHZ-state in QED-cavities we will start from the state

$$|\psi\rangle = \alpha|0\rangle_{A1}|0\rangle_{A2}\dots|0\rangle_{AN} + \beta|1\rangle_{A1}|1\rangle_{A2}\dots|1\rangle_{AN} \quad (7)$$

where $N$ stands for the number of cavities involved.

The cavities $B_1, B_2, \dots, B_N$ are prepared in the vacuum state whereas the cavities $A_1, A_2, \dots, A_N$ are prepared in the state which one wants to teleport. The number of required atoms depends on how many cavity-states one wants to teleport.

The first excited atom ($|e\rangle_1$) is sent to interact resonantly with the cavity $B_1$ leaving the system (cavities $B_1, B_2, \dots, B_N$, and the atom 1) to the state,

$$|\varphi\rangle_1 = \frac{1}{\sqrt{2}}[|0\rangle_{B1}|0\rangle_{B2}\dots|0\rangle_{BN}|e\rangle_1 - i|1\rangle_{B1}|0\rangle_{B2}\dots|0\rangle_{BN}|g\rangle_1],$$

for $gt = \frac{\pi}{4}$. Next, the atom 1 crosses the cavity $A_1$ and in sequence it is detected in the excited state $|e\rangle_1$. In this case the whole system is given by the approximated state

$$|\Phi\rangle_4 \cong [\alpha|0\rangle_{A1}|0\rangle_{A2}\dots|0\rangle_{AN}|0\rangle_{B1}|0\rangle_{B2}\dots|0\rangle_{BN} + \beta|0\rangle_{A1}|1\rangle_{A2}\dots|1\rangle_{AN}|1\rangle_{B1}|0\rangle_{B2}\dots|0\rangle_{BN}],$$

for the adjustment $gt' = \frac{7\pi}{4}$.

The same procedure is applied to the sequence of atom; e.g., for the $N$-th atom crossing the cavity $B_N$ and in sequence the cavity $A_N$, after being detected in the state $|e\rangle_N$ leaves the entire system in the approximated state

$$|\Phi\rangle_6 \cong (\alpha|0\rangle_{B1}|0\rangle_{B2}\dots|0\rangle_{BN} + \beta|1\rangle_{B1}|1\rangle_{B2}\dots|1\rangle_{BN}) \otimes |0\rangle_{A1}|0\rangle_{A2}\dots|0\rangle_{AN},$$

with the adjustment $gt' = \frac{7\pi}{4}$. Then we note that the system composed by the cavities $B_1, B_2, \dots, B_N$ is projected onto the teleported state

$$|\psi\rangle_T = \alpha|0\rangle_{B1}|0\rangle_{B2}\dots|0\rangle_{BN} + \beta|1\rangle_{B1}|1\rangle_{B2}\dots|1\rangle_{BN}, \quad (8)$$

where the cavities $A_1, A_2, \dots, A_N$ remains in the vacuum state.

In Zheng [25] the fidelity depends on the unknown coefficient $C_e$ according to the inequality

$$F = \frac{1/2}{1/2 + |C_e|^2 \cos \sqrt{2} g t'} \geq 0.987. \qquad (9)$$

For the bipartite case we have shown that this dependence is ruled out [8]

$$F = \frac{\sin^2 g t'}{\cos^2 \sqrt{2} g t' + \sin^2 g t'} \simeq 0.97. \qquad (10)$$

In the same way, the fidelity of the state given by the (8) is easily generalized by noting the dependence with the coefficients for the case of odd "$N$". Then, we have

$$\begin{cases} F \cong (0.987)^N, & \text{for } N \text{ even}, \\ F \gtrsim (0.987)^N, & \text{for } N \text{ odd}, \end{cases} \qquad (11)$$

where "$N$" is the number of states describing the cavity-modes which one wants to teleport.

The probability success is given by

$$P = (0.25)^N. \qquad (12)$$

## 4 Conclusion

When comparing our scheme with that by Ikran et al. [15] we observe the experimental advantage of ours: while they have to make $2^{2n}$ measurements in the basis states of $A1, \ldots, An, B1, \ldots, Bn$ system, we need to make only "$n$" atomic measurements. The measurements in Ikran et al. [15] involves $2n$ parameters: $n$ of them concern the phase whereas the remaining ones concern the photon numbers inside the cavities $A1, \ldots, An, B1, \ldots, Bn$. The control of the interaction times, through the Stark field adjustment allows one to easily achieve the times during which the atom is resonant with each selected cavity-mode [9, 23]. To reduce atomic spontaneous decay we have assumed the Rydberg atoms in circular states with principal quantum numbers 50 and 51, the radiative time is $T_r = 3 \times 10^{-2}$ s, and the coupling constant is $g = 2\pi \times 25$ kHz [9, 23]. Negligible cavity loss is also required during the whole process of teleportation. Cavity lifetimes for high-Q cavities should be long enough as all the interactions of atom with cavities should be completed before the cavity dissipation. High-quality factors of such cavities and control of atomic beams during the whole teleportation process may pose limitations on the suggested scheme.

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